Let P be the point $(-1, 1, -3)$ and R be the point $(1, -1, -2)$. **SCORE: _____ / 76 PTS**

Let $\overrightarrow{PQ}$ be the vector $< -1, 4, 1 >$ and $\overrightarrow{PS}$ be the vector $-\vec{i} + 2\vec{k}$.

[a] Find parametric equations of the line which passes through P and R.

$$\overrightarrow{PR} = \langle 1 - -1, -1 - 1, -2 - -3 \rangle = \boxed{\langle 2, -2, 1 \rangle}_{3} = \vec{d}$$

$$\boxed{\begin{aligned} x &= -1 + 2t \\ y &= 1 - 2t \\ z &= -3 + t \end{aligned}} \; 4 \qquad \text{OR} \qquad \begin{aligned} x &= 1 + 2t \\ y &= -1 - 2t \\ z &= -2 + t \end{aligned} \qquad \text{OR OTHER ANSWERS POSSIBLE}$$

3

[b] Find the area of triangle PQR.

$$\frac{1}{2} \| \overrightarrow{PQ} \times \overrightarrow{PR} \| = \frac{1}{2} \| \langle -1, 4, 1 \rangle \times \langle 2, -2, 1 \rangle \| = \frac{1}{2} \| \langle 6, 3, -6 \rangle \|$$

$$= \frac{1}{2} \sqrt{36 + 9 + 36}$$

$$= \frac{1}{2} \sqrt{81}$$

$$= \boxed{\frac{9}{2}} \; 4, 3$$

$$\left| \begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ -1 & 4 & 1 & -1 & 4 \\ 2 & -2 & 1 & 2 & -2 \end{array} \right|$$

$$= 4\vec{i} + 2\vec{j} + 2\vec{k} - (-2\vec{i} - \vec{j} + 8\vec{k}) = \boxed{\langle 6, 3, -6 \rangle} \; 9$$

SANITY CHECK:

$$\langle 6, 3, -6 \rangle \cdot \langle -1, 4, 1 \rangle = \boxed{-6 + 12 - 6 = 0} \checkmark$$

$$\langle 6, 3, -6 \rangle \cdot \langle 2, -2, 1 \rangle = \boxed{12 - 6 - 6 = 0} \checkmark \; 3$$

[c] Find a vector of magnitude 7 in the same direction as $\overrightarrow{PR}$.

$$7 \left(\frac{1}{\|\overrightarrow{PR}\|} \right) \overrightarrow{PR} = 7 \left(\frac{1}{\sqrt{4 + 4 + 1}} \right) \langle 2, -2, 1 \rangle = 7 \left(\frac{1}{\boxed{3}} \right) \langle 2, -2, 1 \rangle = \boxed{\left\langle \frac{14}{3}, \frac{-14}{3}, \frac{7}{3} \right\rangle}$$

4 3 3

[d] Find a unit vector perpendicular to both $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

$$\frac{1}{\|\overrightarrow{PQ} \times \overrightarrow{PR}\|} \overrightarrow{PQ} \times \overrightarrow{PR} = \underbrace{\boxed{\frac{1}{9}}}_{3} \underbrace{\boxed{\langle 6, 3, -6 \rangle}}_{4} = \underbrace{\boxed{\left\langle \frac{2}{3}, \frac{1}{3}, -\frac{2}{3} \right\rangle}}_{3}$$

[e] Find symmetric equations of the line which is parallel to the line in part [a], and which also passes through the point $(0, 5, -3)$.

↳ D.V. PARALLEL → USE SAME $\vec{d}$

3

$$\boxed{\frac{x - 0}{2} = \frac{y - 5}{-2} = \frac{z + 3}{1}} \longrightarrow \boxed{\frac{x}{2} = \frac{5 - y}{2} = z + 3}$$

4 3

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[f] Find the volume of the parallelepiped with $\overrightarrow{PQ}$, $\overrightarrow{PR}$ and $\overrightarrow{PS}$ as adjacent edges.

$$|(\overrightarrow{PQ} \times \overrightarrow{PR}) \cdot \overrightarrow{PS}| = |\underbrace{\langle 6, 3, -6 \rangle \cdot \langle -1, 0, 2 \rangle}_{4}| = |-6 + 0 - 12| = \underbrace{\boxed{18}}_{3}$$

[g] Find the standard (point-normal) equation of the plane which is perpendicular to both the lines in parts [a] and [e] (which are parallel to each other), and which also passes through the point $(-4, 0, 2)$.

↳ N.V. PARALLEL TO D.V. → USE $\vec{n} = \vec{d}$

$$2(x+4) - 2(y) + (z-2) = 0$$

3

$$2(x+4) - 2y + (z-2) = 0$$

4 3

Find all octants in which $xy < 0$ and $xz < 0$ simultaneously (ie. both conditions have to be true at the same time). **SCORE: _____ / 10 PTS**

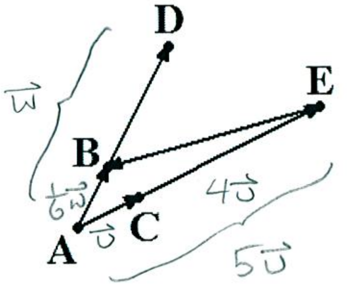
2 $x > 0,\ y < 0,\ z < 0 \longrightarrow O_{4+4} \longrightarrow O_8$ 4

2 $x < 0,\ y > 0,\ z > 0 \longrightarrow O_2$ 2

In the diagram below, ABD and ACE are both line segments. **SCORE: _____ / 10 PTS**

CE is four times the length of AC, and AD is six times the length of AB. **(NOTE: The diagram is <u>NOT</u> drawn to scale.)**

If $\vec{u} = \overrightarrow{AC}$ and $\vec{w} = \overrightarrow{AD}$, find an expression for $\overrightarrow{EB}$ in terms of $\vec{u}$ and $\vec{w}$.



$$\overrightarrow{AE} + \overrightarrow{EB} = \overrightarrow{AB}$$

$$5\vec{u} + \overrightarrow{EB} = \frac{1}{6}\vec{w}$$

$$\overrightarrow{EB} = \underbrace{\boxed{\frac{1}{6}\vec{w}}^{\,3} - \boxed{5\vec{u}}^{\,3}}_{4}$$

If $\vec{u}$ is a vector of magnitude $2\sqrt{3}$, $\vec{v}$ is a unit vector, and $\vec{u} \cdot \vec{v} = 3$, find the angle between $\vec{u}$ and $\vec{v}$. SCORE: _____ / 10 PTS

$$\cos^{-1} \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|} = \cos^{-1} \frac{3}{(2\sqrt{3})(1)} = \underbrace{\boxed{\cos^{-1}}}_{3} \underbrace{\boxed{\frac{\sqrt{3}}{2}}}_{4} = \underbrace{\boxed{\frac{\pi}{6}}}_{3}$$

Fill in the blanks. List all correct answers. **SCORE: _____ / 12 PTS**

$\vec{u}\cdot\vec{u} = \|\vec{u}\|^2 = 63$

[a] If $\vec{u}\cdot\vec{u} = 63$, then $\|\vec{u}\| =$ $\boxed{3\sqrt{7}}$ 3 and $\vec{u}\times\vec{u} =$ $\boxed{\vec{0}}$ 2.

[b] The equation of the $x-$axis is $\boxed{y = z = 0}$ 2 and the equation of the $xy-$plane is $\boxed{z = 0}$ 2.

$\langle x-4, y+1, z+8\rangle = \langle 0, 3, -2\rangle$

[c] If the initial point of $\vec{v} = 3\vec{j} - 2\vec{k}$ is $(4, -1, -8)$, then the terminal point of $\vec{v}$ is $\boxed{(4, 2, -10)}$. 3

Consider the sphere $x^2 + y^2 + z^2 + 6x + 10y - 12z + 21 = 0$. **SCORE: _____ / 20 PTS**

[a] Find the center and radius of the sphere.

$$x^2+6x+9 \quad 1\tfrac{1}{2} \quad + \quad y^2+10y+25 \quad 1\tfrac{1}{2} \quad + \quad z^2-12z+36 \quad 1\tfrac{1}{2} \quad = -21+9+25+36 \quad 1\tfrac{1}{2}$$

$$(x+3)^2+(y+5)^2+(z-6)^2=49 \quad 1$$

CENTER $(-3,-5,6)$ $1\tfrac{1}{2}$

RADIUS $\sqrt{49}=7$ $1\tfrac{1}{2}$

[b] Find the equation of the yz – trace. Describe the yz – trace as specifically as possible.

$$(0+3)^2+(y+5)^2+(z-6)^2=49$$

$$2 \quad (y+5)^2+(z-6)^2=40$$

2 CIRCLE CENTER $(0,-5,6)$ 2

RADIUS $\sqrt{40}=2\sqrt{10}$ 2

IN YZ-PLANE 2

If $\vec{u} = <9, -3, c+1>$ is perpendicular to $\vec{v} = <c, -8, c>$, find all possible values of c. **SCORE: _____ / 12 PTS**

$$\vec{u} \cdot \vec{v} =_{4} \underline{9c + 24 + c^2 + c} = \underline{0} \quad 3$$

$$\underline{c^2 + 10c + 24 = 0} \quad 2$$

$$(c+4)(c+6) = 0$$

$$\underline{c = -4, -6} \quad 3$$